Survey Designs for Distance Sampling: A Study of Zebra Mussels

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Carleton College

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Agenda

- Statistical Reasoning

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- Lake Burgan Data

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- Simulations

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- Time Analysis

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- Lake Burgan Data
- Simulations
- Time Analysis
- Further Research

Estimating Mussel Abundance and Density

2 Step Approach:

1. Fit a detection function, $g(x)$, to our data

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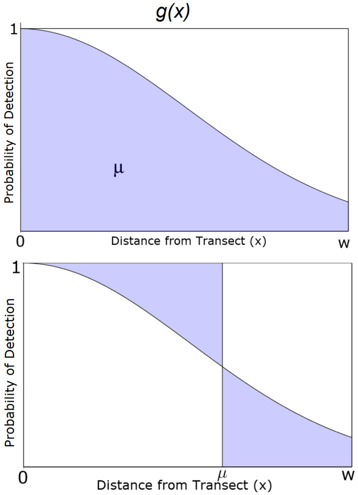
- We used a half-normal distribution for our models where

$$g(x) = exp\left[\frac{-x^2}{2\sigma^2}\right]$$

Estimating Detection Parameters

- Need proper probability density function that integrates to 1 for MLE

$$f(x) = \frac{g(x)}{\mu}$$



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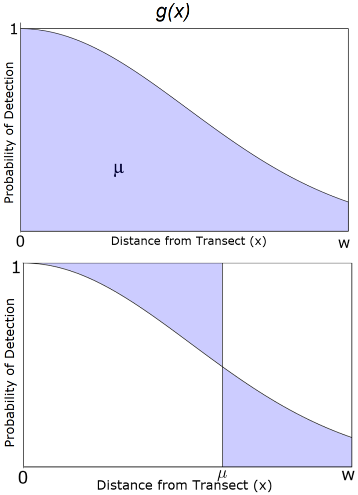

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Estimating Detection Parameters

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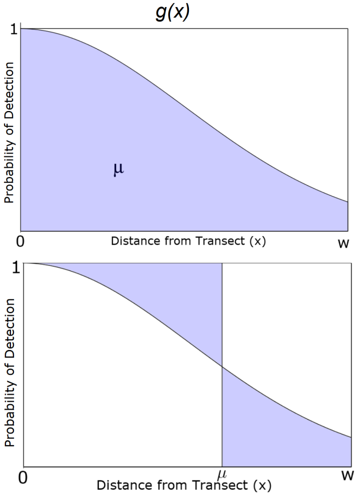
$$f(x) = \frac{g(x)}{\mu}$$

- Normalizing Constant μ
  - Effective Half-Width

$$\mu = \int_0^w g(x)dx$$



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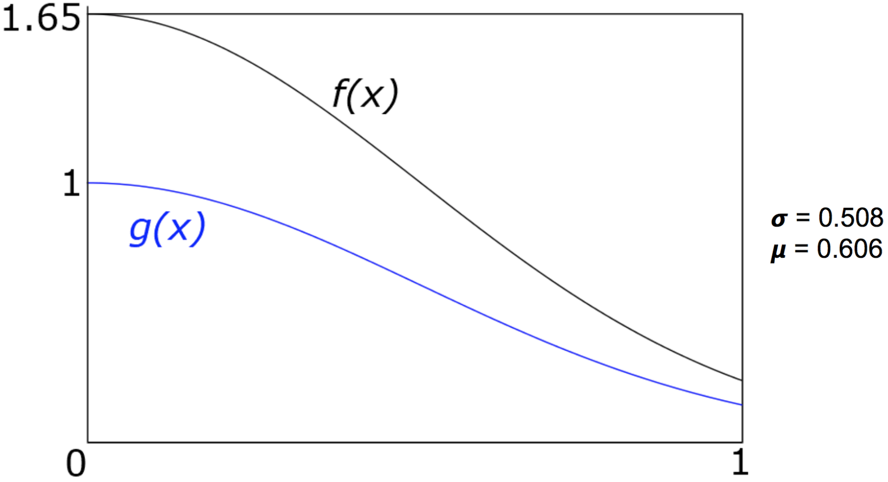

Estimating Detection Parameters

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  - Likelihood Function

$$L_x = \Pi_{i=1}^{n} f(x_i) = \frac{\Pi_{i=1}^{n} g(x_i)}{\mu^n} = \mu^{-n} exp\left[\frac{-\sum_{i=1}^{n} x_i^2}{2\sigma^2}\right]$$

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- Find σ that maximizes L_x

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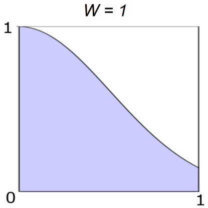
  - Only when we assume $w = \infty$

Average Probability of Detection

- Average Detectability

$$P_a = \frac{2\mu L}{2wL} = \frac{\mu}{w}$$



- $P_a = \mu = 0.606$
- $\sigma = 0.508$

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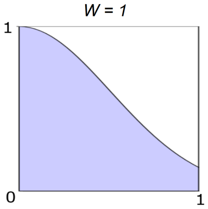

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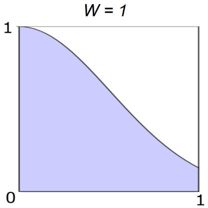

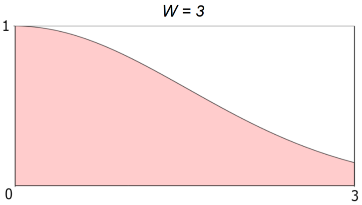

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- And plugging back in we have

$$\hat{N} = \frac{nA}{a\hat{P}_a}$$

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- Thus, rewritten

$$SE(\hat{D}) = \hat{D} * CV(\hat{D})$$

Calculating Standard Error of Density

- Thus, we write

$$CV(\hat{D}) = \sqrt{\frac{\frac{K}{L^2(K-1)}\sum_{k=1}^{K} l_k^2(\frac{n_k}{l_k} - \frac{n}{L})^2}{(n/L)^2} + \frac{1}{2n}}$$

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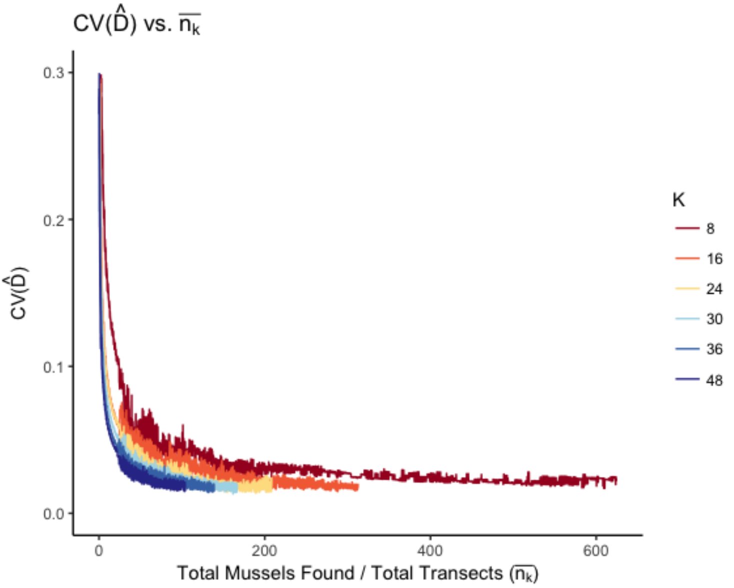
- n_k = number of mussels found on the kth transect
- l_k = length of of the kth transect (30 meters for all)

Effects of Changing K and n



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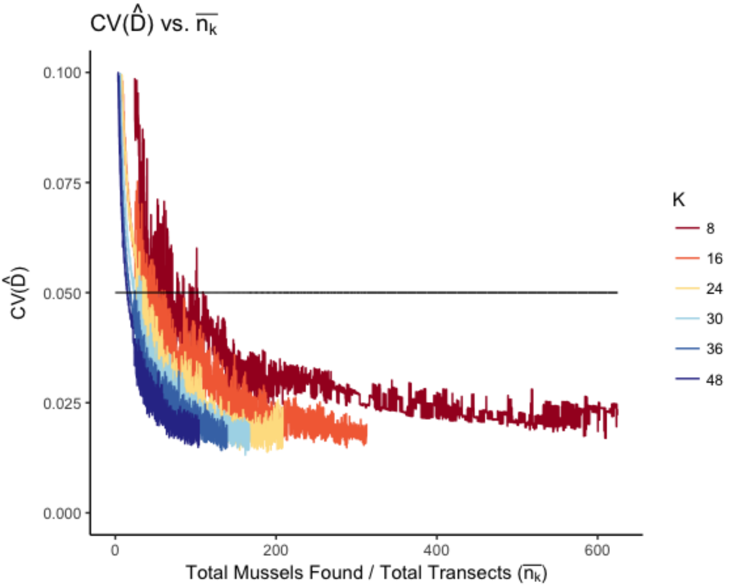

Cause of Variability in cv($\hat{D}$)

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- Multinomial randomization for variation in transects
- Assume equal probability

Lake Burgan


Google
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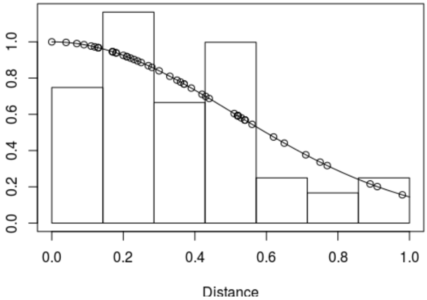
Fitting Lake Burgan Data to a Model

- $n = 52$ mussels



Parameter	Estimate	Std. Error	CV
$\hat{\sigma}$	0.508	0.084	0.165
$\hat{\mu} = \hat{P}_a$	0.606	0.075	0.123
$\hat{D}$	0.090	0.0199	0.222
$\hat{N}_a$	89.584	19.921	0.222
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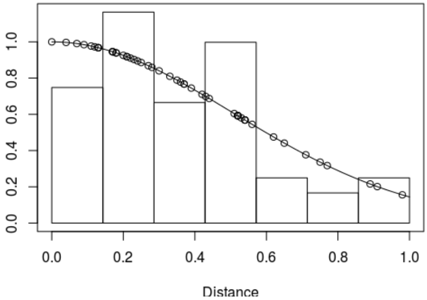

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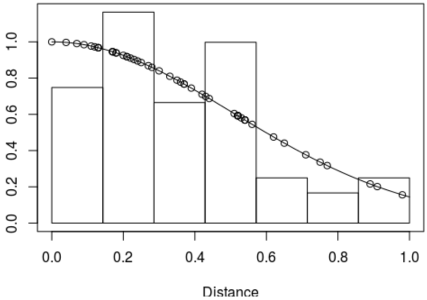

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The variables we controlled in our simulations were:

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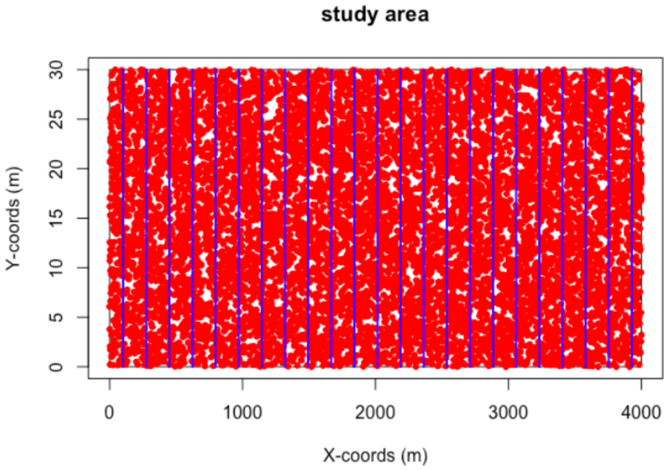
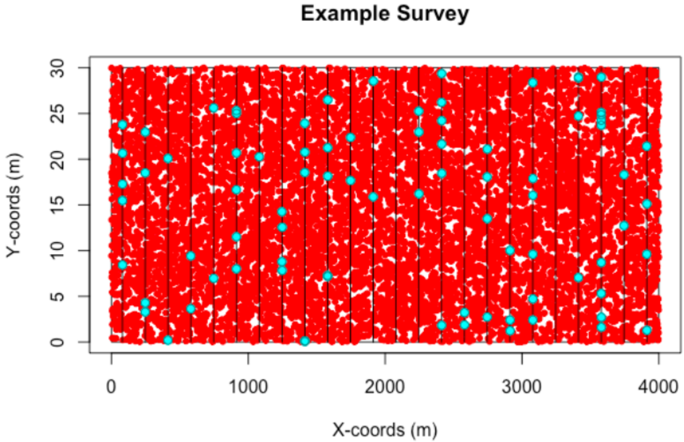
- Number of Transects K
- Detection Scale Parameter σ
- Number of Strata
- Addition of Hotspots (areas of elevated density)

Basic Simulation



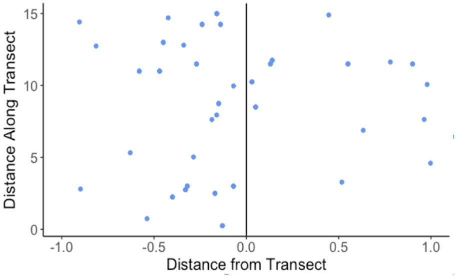
N = 10,000
K = 24

Basic Simulation



N = 10,000
K = 24
σ = .7

How the Simulation Works



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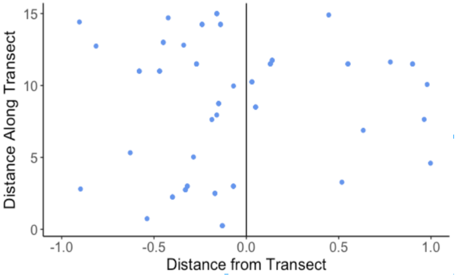

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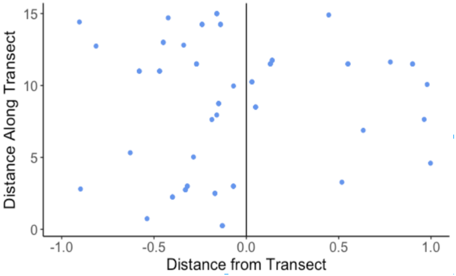

- x = distance from transect
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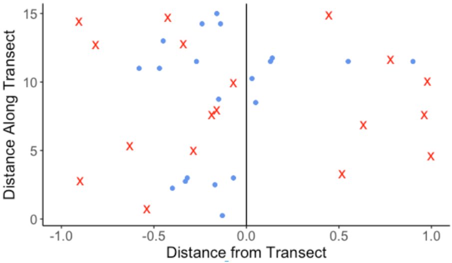

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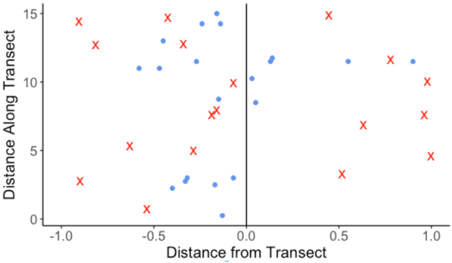

- Detection $\sim$ Bernoulli(p_i)
- Assigned a 1 if found, 0 if not found (red X)

Comparing Simulation Results

There are two results we use to quantify the difference between sampling designs:

- Percent Bias (Accuracy)

$$\% \hat{Bias}_{\hat{N}} = \frac{\hat{\hat{N}} - N}{N} \times 100\%$$

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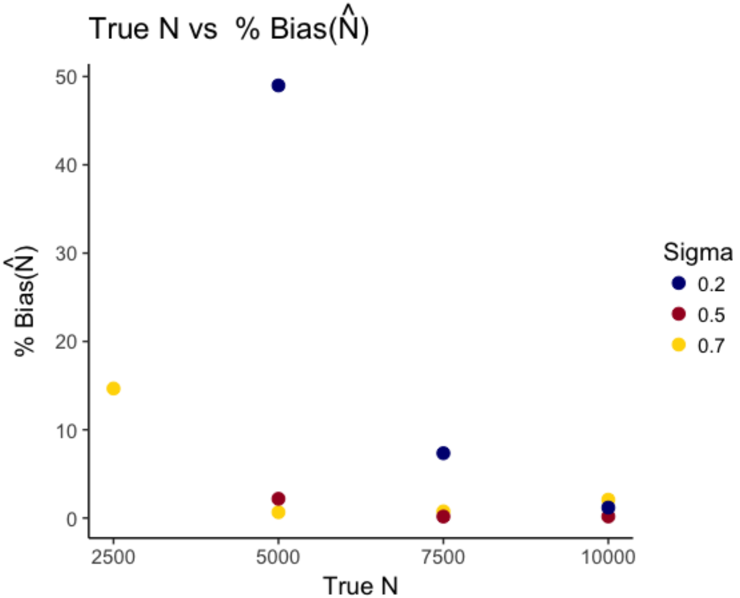
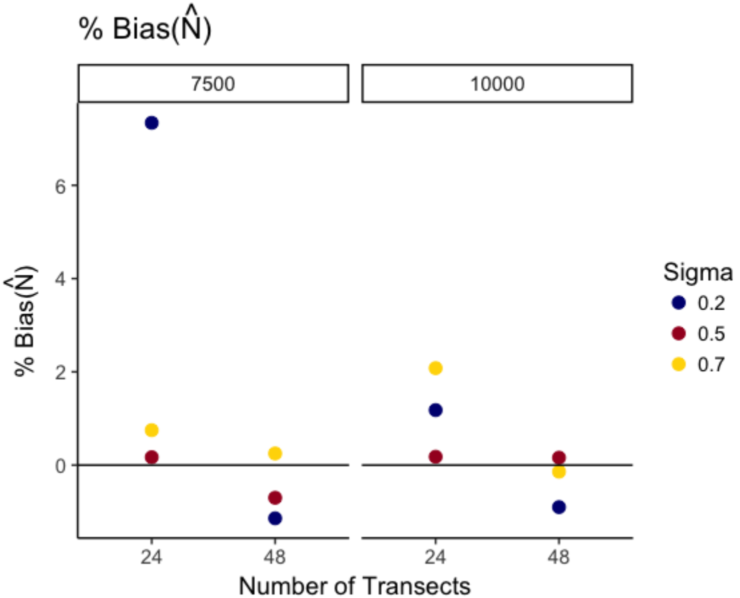
- Coefficient of Variation (Precision)

$$CV(\hat{N}) = \frac{SE(\hat{N})}{\hat{\hat{N}}}$$

Varying N and σ



Varying number of transects, K



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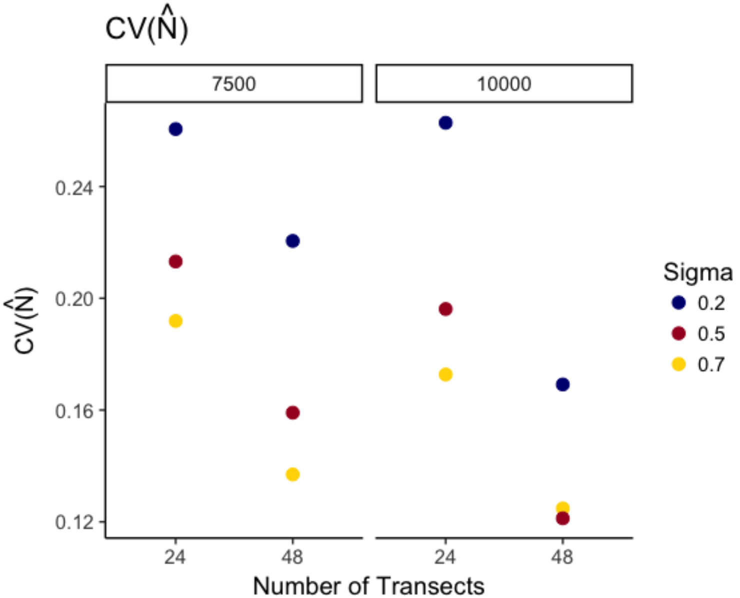
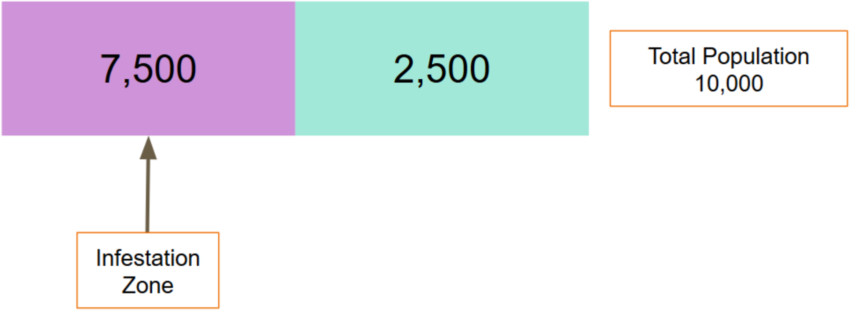
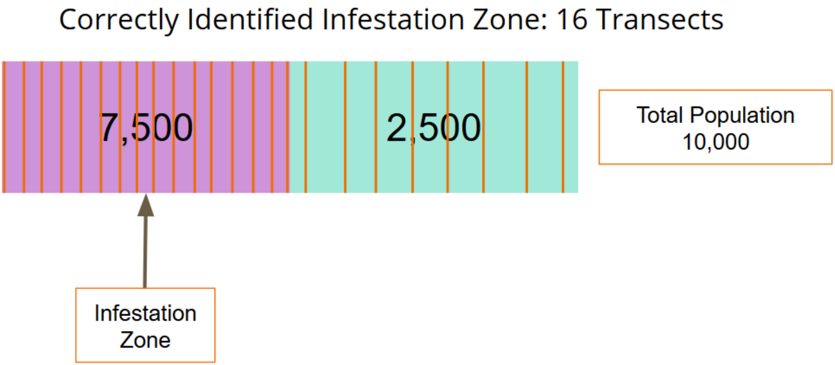


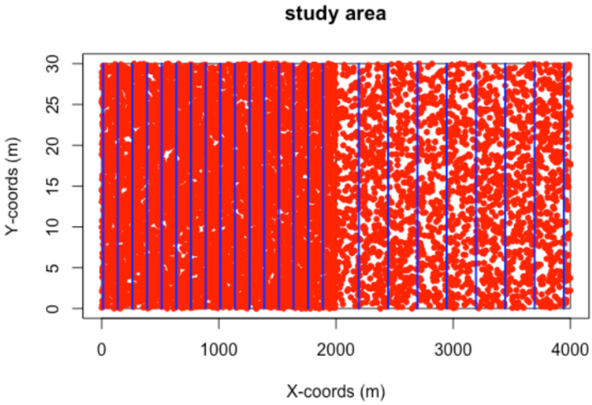
Stratified Design



Stratified Design: Correctly Identified

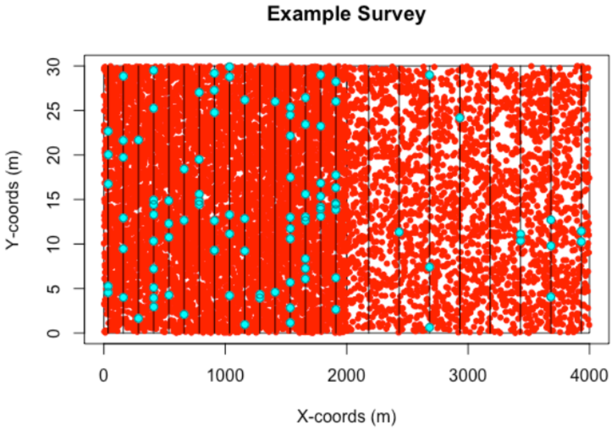


Stratified Design Simulation



N = 7,500 & 2,500
K = 16, 8

Stratified Design Simulation



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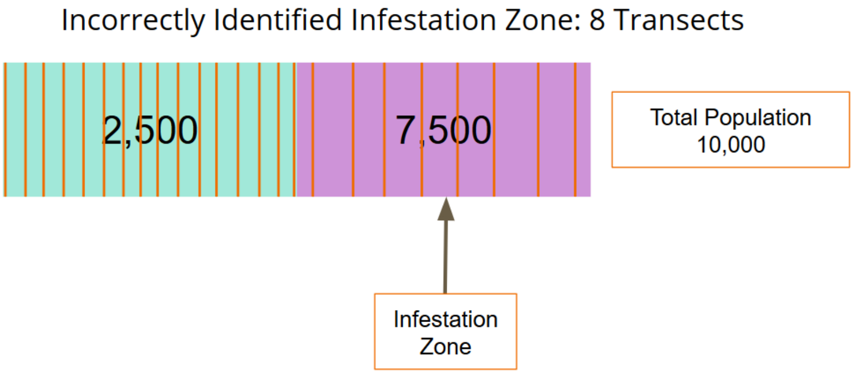
Stratified Design: Incorrectly Identified



Stratified Design Results

Table: How Stratified Designs Effect Estimates

Design	$\bar{n}$	$\hat{\bar{N}}$	$\%Bias_{\hat{N}}$	$CV(\hat{N})$
Constant K	90	10,107	1.07%	.16
Correctly Identified	105	10,032	.32%	.16
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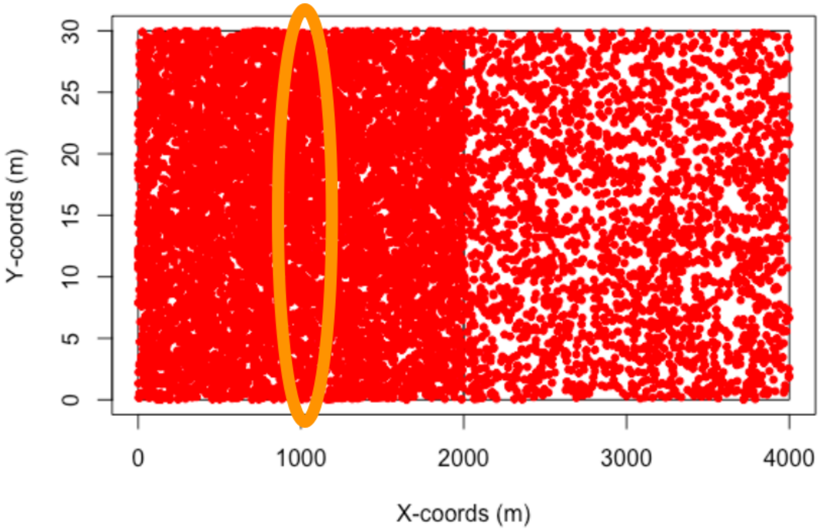
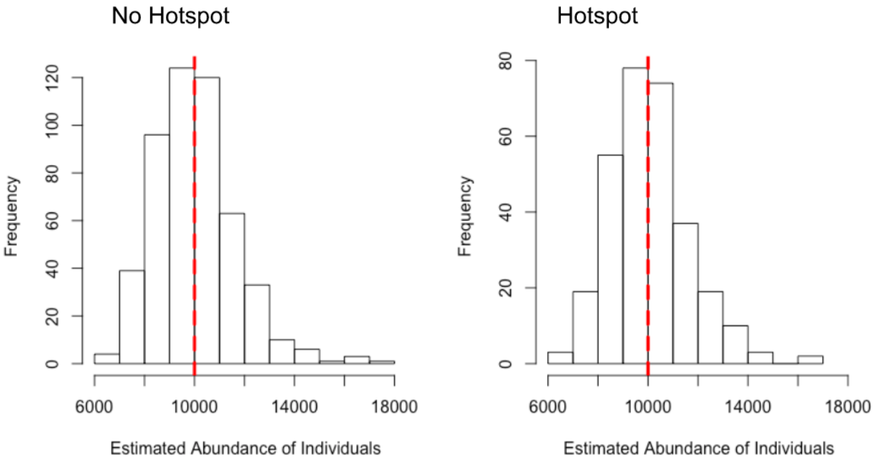
A .01 difference in $CV(\hat{N})$ is a difference in SE of $.01 * 10000 = 100$ mussels

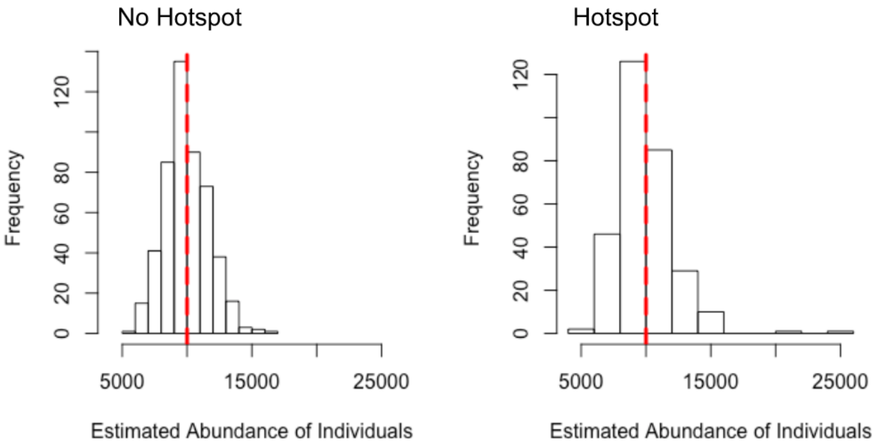
Addition of a Hotspot



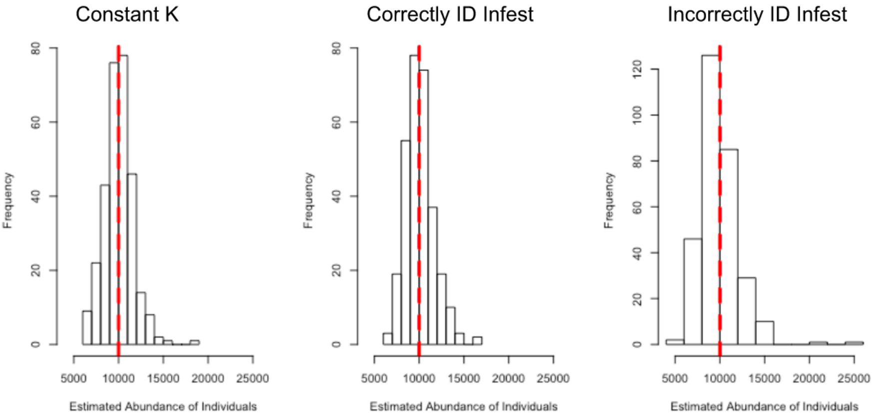
Hotspot Results:Correctly Identified Infestation Zone



Hotspot Results: Incorrectly Identified Infestation Zone



Simulation Results: Infestation Zone with Hotspot



$CV(\hat{N}) = 0.163$ $\quad$ $CV(\hat{N}) = .155$ $\quad$ $CV(\hat{N}) = .222$

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  - $\%Bias_{\hat{N}}$ was not significantly different than 0
- Incorrectly identified hotspots can create large prediction errors

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- Buckland suggests an n of at least 60-80
  - $\%Bias_{\hat{N}}$ was not significantly different than 0
- Incorrectly identified hotspots can create large prediction errors
- Predicted $SE(\hat{N})$ was smaller than the actual distribution of the $\hat{N}$ values from the 300-500 runs

Simulation Discussion

- Higher $\bar{n}$ meant more accurate and precise results
  - Greater N and σ increase n
- Buckland suggests an n of at least 60-80
  - $\%Bias_{\hat{N}}$ was not significantly different than 0
- Incorrectly identified hotspots can create large prediction errors
- Predicted $SE(\hat{N})$ was smaller than the actual distribution of the $\hat{N}$ values from the 300-500 runs
  - $SE(\hat{N})$ equation is biased

Experiment on Time

- Randomly placed 30 small marshmallows within transect

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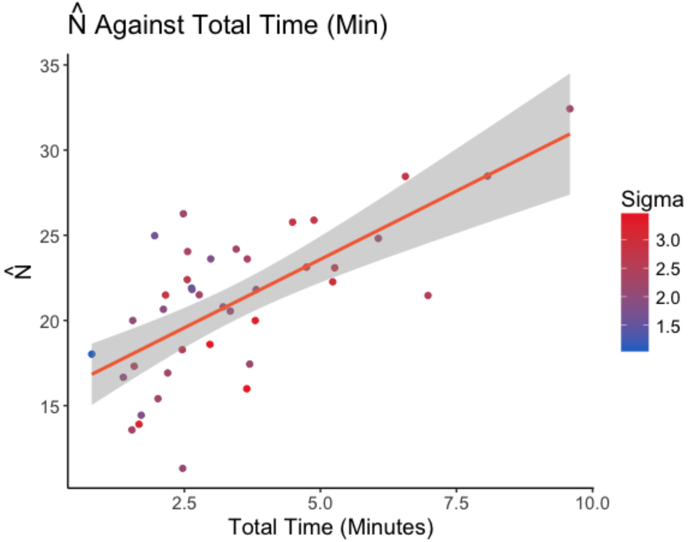
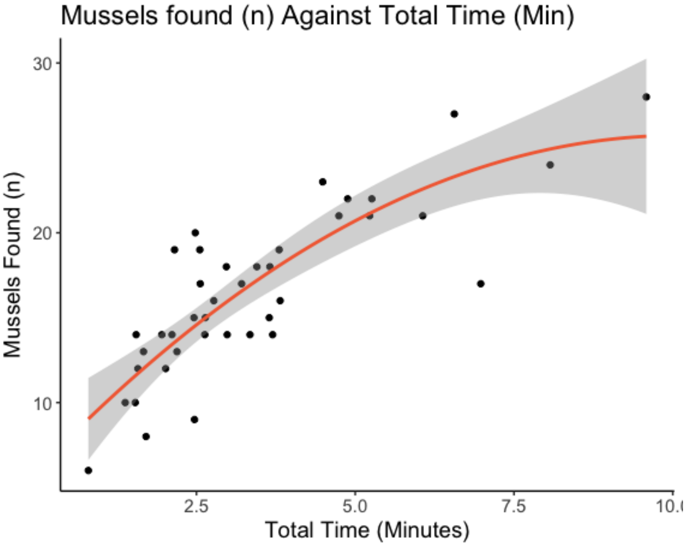
- Timed participants to see how time affects estimates

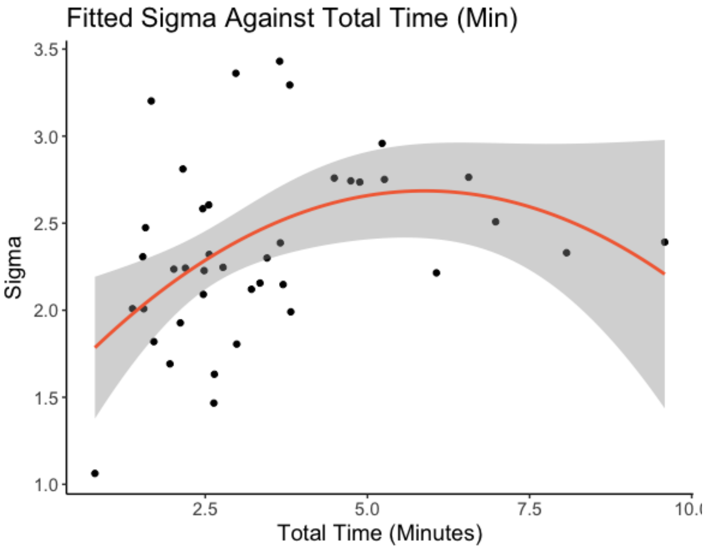
$\hat{N}$ Against Time



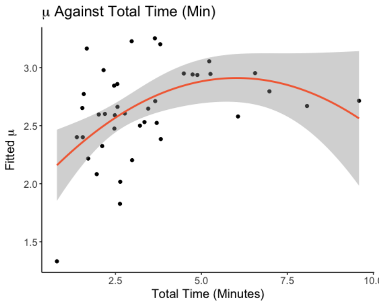
n Against Time

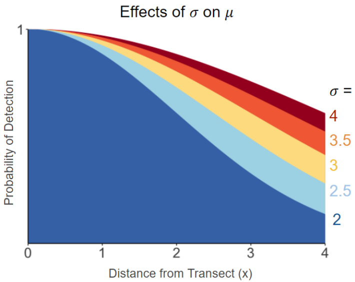


σ Against Time



Fitted μ


μ Against Total Time (Min)
Fitted μ
3.0
2.5
2.0
1.5
2.5
5.0
7.5
Total Time (Minutes)


Effects of σ on μ
Probability of Detection
1
0
1
2
3
4
Distance from Transect (x)
σ =
4
3.5
3
2.5
2

Relationship between σ, μ, n, and $\hat{N}$

$$\hat{N} = \frac{nA}{a\hat{P}_a}$$

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$\hat{N}$ is a function of n and μ, which depends on σ

Experiment Takeaways

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- Time has a nonlinear relationship with σ, μ, and n
- Time has a linear relationship with $\hat{N}$ as a result
- Choose a time that maximizes detection
- Choose a time that optimizes σ
- Increased σ implies increased n
- Supports the claim that we can control $CV(\hat{D})$ using n

Further Research

- Incorporating habitat covariates

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References

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